

Local field modulated entanglement among three distant atoms

Yan-Qing Guo*, Jing Chen, He-Shan Song[†]
 Department of Physics, Dalian University of Technology
 Dalian, 116024, P. R. China

February 1, 2008

Abstract

We extend the scheme for that proposed by S. Mancini and S. Bose (Phys. Rev. A **70**, 022307(2004)) to the case of triple-atom. Under mean field approximation, we obtain an effective Hamiltonian of triple-body Ising-model interaction. Furthermore, we stress on discussing the influence of the existence of a third-atom on the two-atom entanglement and testing the modulation effects of locally applied optical fields and fiber on the entanglement properties of our system.

PACS number: 03.67.-a, 03.67.-Hz, 42.81.Qb

1 Introduction

Generating and maintaining the entanglement between two spatially separated atoms plays an important role in many quantum information processing, such as quantum storage[1], quantum key distribution[2] and quantum states swapping[3], since need no transmitting real particles to distance. To efficiently entangle two distant atoms, one must create some kind of interaction that is off-diagonal in its energy eigenstates representation. Generally, such interaction is equivalent to exchange real photons between two spatially related atoms. Many schemes have been proposed to realize this kind of entanglement[4-13]. Most of these schemes involve two cavities in each of which an atom is trapped. In Ref. [4], the author proposed such a scheme. He showed how an effective direct interaction between two atoms is established. The author used photon as an intermediate quantum information carrier to map the quantum information from

*E-mail: guoyqthp@yahoo.com

[†]Corresponding author: hssong@dlut.edu.cn

an atom in one cavity to an atom in another cavity. By eliminating the optical fields ingoing and outgoing the local cavity, the author found that two distant atoms can directly interact in a form of Ising spin-spin coupling. It is meaningful conceptually in quantum measurement and testing Bell's inequalities and engineeringly in quantum encryption[14] and constructing universal quantum gate[15] that are essential elements for designing quantum network. While, in realizing quantum network, an atomic ensemble is usually required[1]. The existence of collective interaction inevitably influence the entanglement shared by atoms. In this paper, we extend the model in Ref. [4] to triple-atom situation. We focus on investigating the affect of the existence of the third atom on the entanglement properties and the modulation of operating local optical field plus fiber on the information signal. This system can be easily extended to N-atom system.

2 Optical Fiber Connected Three-Atom-Cavity System

Fig. 1 shows the imagined triple-atom composed setup. Two-level atoms A , B and C locate in spatially distant cavities 1, 2 and 3 respectively. Cavities 1 and 3 are assumed to be single-sided cavities, cavity 2 to be double-sided cavity. An off-resonant driving external field A is added on cavity 1. In each cavity, a local laser field which is resonantly coupled to the local atom is applied. Two neighbouring cavities are connected with optical fiber.

The Hamiltonian for the global system can be written as

$$H = H_{atom} + H_{field} + H_{bath} + H_{int1} + H_{int2}. \quad (1)$$

In the interaction picture, The Hamiltonian should include the interactions between atom and bath and that between field and bath

$$H_{int} = H_{in1} + H_{int2}, \quad (2)$$

where[16]

$$H_{int1} = \chi A^\dagger A \sigma_1^z + \chi B^\dagger B \sigma_2^z + \chi C^\dagger C \sigma_3^z, \quad (3)$$

and[17]

$$\begin{aligned}
H_{int2} = & i \int_{-\infty}^{+\infty} d\omega \{ \kappa_A [b_A(\omega)A^\dagger + h.c.] + \kappa_{B,L} [b_{B,L}(\omega)B^\dagger + h.c.] \\
& + \kappa_{B,R} [b_{B,R}(\omega)B^\dagger + h.c.] + \kappa_C [b_C(\omega)C^\dagger + h.c.] \}, \quad (4)
\end{aligned}$$

where $A(A^\dagger)$, $B(B^\dagger)$ and $C(C^\dagger)$ the field annihilation (creation) operators in cavities A, B and C, respectively, $\chi = g^2/\Delta$ with g the dipole coupling strength of atom on field and Δ the detuning of field from the atomic internal transition, $\sigma_i^z (i = 1, 2, 3)$ represents the particle inversion number of atom i .

The kinetic equations for the field operators turn out to be[17]

$$\begin{aligned}
\dot{A} &= -i\Delta A - i\chi A\sigma_1^z - \frac{\gamma_A}{2}A + \sqrt{\gamma_A}A_{in} + A, \\
\dot{B} &= -i\Delta B - i\chi B\sigma_{2z} - \frac{\gamma_{B,L} + \gamma_{B,R}}{2}B + \sqrt{\gamma_{B,L}}B_{in,L} + \sqrt{\gamma_{B,R}}B_{in,R}, \\
\dot{C} &= -i\Delta C - i\chi C\sigma_{2z} - \frac{\gamma_C}{2}C + \sqrt{\gamma_C}C_{in}, \quad (5)
\end{aligned}$$

where $\gamma_i = 2\pi k_i^2(\omega)$, A is the amplitude of the input off-resonant driving field in cavity A.

If cavities A, B and cavities B, C are connected by fiber (as shown in Figure 1), the input-output condition should be included, such that[18]

$$\begin{aligned}
\dot{A} &= -\frac{\gamma_A}{2}A + \sqrt{\gamma_A}B_{out,L}e^{i\phi_{12}}, \\
\dot{B} &= -\frac{\gamma_{B,L}}{2}B + \sqrt{\gamma_{B,L}}A_{out}e^{i\phi_{21}}, \\
\dot{B} &= -\frac{\gamma_{B,R}}{2}B + \sqrt{\gamma_{B,R}}C_{out}e^{i\phi_{23}}, \\
\dot{C} &= -\frac{\gamma_C}{2}C + \sqrt{\gamma_C}B_{out,R}e^{i\phi_{32}}. \quad (6)
\end{aligned}$$

Assuming all the decay rates $\gamma_A = \gamma_{B,L} = \gamma_{B,R} = \gamma_C = \gamma_0$ and taking into account the usual boundary conditions[17]

$$A_{out}(B_{out,L}, B_{out,R}, C_{out}) + A_{in}(B_{in,L}, B_{in,R}, C_{in}) = \sqrt{\gamma_0}A(B, B, C), \quad (7)$$

we can rewrite the kinetic equations for field operators as

$$\begin{aligned}
\dot{A} &= -(i\Delta + \gamma_0)A - i\chi A\sigma_1^z + \sqrt{\gamma_0}A_{in} + \gamma_0 A_{in} + e^{i\phi_{12}}(\gamma_0 B - \sqrt{\gamma_0}B_{in,L}) + \Lambda, \\
\dot{B} &= -(i\Delta + \gamma_0)B - i\chi B\sigma_2^z + \sqrt{\gamma_0}(B_{in,L} + B_{in,R}) + e^{i\phi_{21}}(\gamma_0 A - \sqrt{\gamma_0}A_{in}) \\
&\quad + e^{i\phi_{23}}(\gamma_0 C - \sqrt{\gamma_0}C_{in}), \\
\dot{C} &= -(i\Delta + \gamma_0)C - i\chi C\sigma_3^z + \sqrt{\gamma_0}C_{in} + e^{i\phi_{32}}(\gamma_0 B - \sqrt{\gamma_0}B_{in,R}). \tag{8}
\end{aligned}$$

The phase factors ϕ_{12} , ϕ_{21} , ϕ_{23} and ϕ_{32} are the phase delay caused from the photon transmission along the optical fiber. Physically, they depend on the frequency of the photons and the distance between cavities. The upper equations are non-linear ones since there exist cross terms include field operators multiplying atomic spin operators. To solve the kinetic equation of fields operators explicitly, these terms must be modified. To do this, we make such assumptions: we are interested in the stationary quantum effects of the field-atom system; strong leakage condition for the cavity; large detuning from the atomic internal transition (see the assumption before). Thus, only the terms besides the cross ones have apparent large gradients. The field operators multiplied by atomic spin operators can be approximately replaced by their stationary values which can be obtained through

$$\frac{d\langle A \rangle}{dt} = \frac{d\langle B \rangle}{dt} = \frac{d\langle C \rangle}{dt} = 0. \tag{9}$$

We get

$$\begin{aligned}
\alpha &= \frac{\Lambda[M(M + \gamma_0) - \gamma_0^2 e^{i(\phi_{23} + \phi_{32})}]}{M^2(M + \gamma_0) - M\gamma_0[e^{i(\phi_{12} + \phi_{21})} + e^{i(\phi_{23} + \phi_{32})}]}, \\
\beta &= \frac{M\alpha - \Lambda}{\gamma_0 e^{i\phi_{12}}}, \quad \gamma = \frac{\gamma_0 e^{i\phi_{32}}\beta}{M}. \tag{10}
\end{aligned}$$

Now, the nonlinear differential Equ. [5] are linearized into

$$\begin{aligned}
\dot{a} &= -(i\Delta + \gamma_0)a - i\chi\alpha\sigma_1^z + \sqrt{\gamma_0}a_{in} + e^{i\phi_{12}}(\gamma_0 b - \sqrt{\gamma_0}b_{in,L}), \\
\dot{b} &= -(i\Delta + \gamma_0)b - i\chi\beta\sigma_2^z + \sqrt{\gamma_0}(b_{in,L} + b_{in,R}) + e^{i\phi_{21}}(\gamma_0 a - \sqrt{\gamma_0}a_{in}) \\
&\quad + e^{i\phi_{23}}(\gamma_0 c - \sqrt{\gamma_0}c_{in}), \\
\dot{c} &= -(i\Delta + \gamma_0)c - i\chi\gamma\sigma_3^z + \sqrt{\gamma_0}c_{in} + e^{i\phi_{32}}(\gamma_0 b - \sqrt{\gamma_0}b_{in,R}). \tag{11}
\end{aligned}$$

where we have replaced field operators A , B and C with $a + \alpha$, $b + \beta$ and $c + \gamma$. Equ. [11] can be easily solved with Laplace transform method. In solving Equ. [11], we can adiabatically eliminate the effect of vacuum input noise on the cavity field operators. The resulting field operators are simply represented by linear combinations of atomic spin operators σ_1^z , σ_2^z and σ_3^z . Substituting the field operators into the interaction Hamiltonian, we get the effective Hamiltonian of the global system in the interaction picture as

$$H_{eff} = J_{12}\sigma_1^z\sigma_2^z + J_{23}\sigma_2^z\sigma_3^z + J_{31}\sigma_3^z\sigma_1^z, \quad (12)$$

which is a triple-body Ising type interaction, similar with that in Ref. [4]. We have defined

$$\begin{aligned} J_{12} &= \gamma_0\chi^2 \text{Im} \{ \alpha\beta^* e^{i\phi_{21}} / [M^2 - W^2] \}, \\ J_{23} &= \gamma_0\chi^2 \text{Im} \{ \alpha\beta^* e^{i\phi_{32}} / [M^2 - W^2] \}, \\ J_{31} &= \gamma_0\chi^2 \text{Im} \{ \gamma_0\gamma\alpha^* [e^{i\phi_{23}} + e^{i\phi_{12}}] / [M(M^2 - W^2)] \}, \end{aligned} \quad (13)$$

with $M = i\Delta + \gamma_0$ and $W^2 = \gamma_0^2 [\frac{1}{4} + e^{i(\phi_{21} + \phi_{12})} + e^{i(\phi_{32} + \phi_{23})}]$. J_{12} and J_{23} are the nearest-neighbour atom radiation pressure, while J_{31} presents next-neighbour atom interaction strength. They all turn out to be zero when cavity fields are resonant with atoms and three cavities are spatially much close to each other.

We have neglected local self-energy terms including σ_i^z by choosing appropriate detuning Δ and self-interaction terms that do not change the initial system state in deriving H_{eff} . Also, we eliminated high order terms that include $\sigma_1^z\sigma_2^z\sigma_3^z$ since the corresponding coupling strength is much weaker than J_{12} , J_{23} and J_{31} . In next section, laser fields are employed to induce two-atom entanglement.

3 Fiber Plus Laser Field Modulated Two-Atom Entanglement

In Ising model, entanglement can be generated between neighbouring spin sites if each of them is coupled with a magnetic field that do no parallel the spin z axis[19]. Note that the Hamiltonian in Equ. [12] can not generate any entanglement because it is diagonal in the $2 \otimes 2 \otimes 2$ dimensions atomic subspace. It

is necessary to add a laser field in each cavity which is resonantly interacts with the local atom, so that the additional Hamiltonian is given by

$$H_{add} = \Gamma \sum_i (\sigma_i + \sigma_i^\dagger), \quad (14)$$

where Γ represents the coupling strength of the laser field and the local atom, i denotes atom. The global Hamiltonian is the sum of two terms $H_{glob} = H_{eff} + H_{add}$. In fact, the existence of resonant laser field would take influence on the effective Hamiltonian when we take some approximations. In order to keep the validity of deriving H_{eff} , the coupling between local atom and the laser field must be assumed much weak. Certainly, this can be easily assured by weakening the amplitude of the laser field or localizing the atom in the cavity since the coupling rate depends on the atom position as $\sin(k_0 z) \exp[-(x^2 + y^2)/\omega_0^2]$ [20].

We are interested in the two-atom subsystem entanglement which is widely used in quantum key distribution and quantum encoding. Wootters proposed a general measurement of two-qubit entanglement which is named as Concurrence [21]

$$C = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}, \quad (15)$$

where λ_i are the none-negative square roots of the four eigenvalues of non-Hermitian matrix $\rho\tilde{\rho}$ with $\tilde{\rho}$ defined as $(\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$.

We depict the two-atom entanglement situation in Fig. 2-4 in different parameter spaces $(J_{12}, J_{23}, J_{31}, B)$. In Fig. 2, we adopt $(4, 4, 0, 0.1)$ for dotted line and $(4, 4, 0.5, 0.1)$ for solid line, then plot the entanglement versus time. Firstly, we investigate the impact of next nearest-neighbour (NNN) atom pair on the entanglement of nearest-neighbour (NN) atom pair. We see from Fig. 2 that, the existence of NNN coupling damages the maximal entanglement of NN atom pair. Fortunately, this coupling can be assigned to tend to zero so that it can be neglected. In doing so, the phase factor ϕ_{21} and ϕ_{32} is assumed to be equal, and phase factor ϕ_{31} is properly choosed so that J_{31} is so much small compares with J_{12} and J_{23} . This can be achieved by selecting special optical fiber. Moreover, NNN coupling does not perturb the period of NN atoms entanglement. So, in utilizing two-atom entanglement, though the probability of entangling maybe affected under different fiber devices, the controlling time of maximal entanglement that is much important in many quantum information experiments [22] should not be changed.

Because of the diversity of fiber path, the phase shift ϕ_{21} and ϕ_{32} could be easily perturbed. In Fig. 3, we investigate the influence of nonuniform coupling on the NN entanglement. The parameter space is (4, 4.1, 0, 0.1) with dotted line for atoms 1-2 and solid line for atoms 2-3. Under this circumstance, three points should be stated: the general two-atom entanglement is reduced compared with that in Fig. 2; there appears a partial pressure effect for series connection, the NN atoms that have stronger coupling share larger entanglement; the general entanglement period is restricted, thus the controlling time for maximal entanglement is compressed. So, in constructing physical quantum information channel for network, the fiber used from point (atom-cavity) to point (atom-cavity) is preferred to be identical even including dissipation on the fiber.

In Fig. 4, we adopt larger Γ than in Fig. 2-3 to study the influence of the interaction between each atom and the local laser field on the NN atom entanglement. Once the data bus constituted by fiber is constructed, the information channel will not be easily improved. While, Fig. 4 indicates that, the alternating of Γ remarkably changes the NN atom entanglement. The period is further depressed, but the amount of entanglement is much enhanced compares with the former results. Under the condition of small Γ , as is pointed before, changing Γ is in fact modulating the entanglement between NN atoms. What we paying attention to is entanglement signals can be manually controlled through alternating Γ . This kind of modulation effects of optical field on atoms or magnetic field on spins have already been concerned[23]. Certainly, the synchronization of Γ s in each local cavity is important for modulating the entanglement, the related discussion will be involved elsewhere.

4 Conclusion

We have discussed the preparation of effective two-atom entanglement in a fiber connected triple-atom system. Using input-output theory for lossy cavity, after tracing over the field variables, we treated the stationary system as a triple-body Ising problem. The inversion of the particle number for two atomic levels was thus analogous with the polarization of electron spin. Local laser fields were applied to generate and modulate the entanglement of two atoms. The fiber connected atoms is in fact a kind of data bus for such a simple multi-atom network, the general influence of which on the two-atom entanglement

was studied. The phase shift of the photons in the fiber has been found to determine the interaction strength between two Ising-kind sites. In addition, the dissipation of the photon information through the fiber should be investigated, while, the impact of dissipation can also be included in the interaction strength but acts as a decaying exponential factor of the length of the fiber, in the form of $e^{-\nu L}$, with ν the dissipation per unit length, L the total length of the fiber. The local laser field plays a key role in modulating the entanglement properties. So, making an opportune choice between fiber and local laser field is optimal in generating such entanglement. Certainly, a practically physical quantum network should be constituted by a large number of atoms[1, 24]. Our system can be extended to one-dimension atomic chain or higher dimension systems. Thus, they may turn out to be a candidate for constructing quantum information data-bus.

Acknowledgement 1 *This work is supported by NSFs of China, under Grant No. 10347103, 10305002 and 60472017.*

References

- [1] C. H. van der Wal, et al., Science **301**, 196(2003); A. Kuzmich, et al., Nature **423**, 731(2003).
- [2] K. Inoue, E. Waks, and Y. Yamamoto, Phys. Rev. Lett. **89**, 037902(2002); R. J. Hughes, G. L. Morgan, and C. G. Peterson, J. Mod. Opt. **47**, 533(2000).
- [3] A. Kuzmich and E. S. Polzik, Phys. Rev. Lett. **85**, 5639-5642 (2000).
- [4] S. Mancini and S. Bose, Phys. Rev. A **70**, 022307(2004).
- [5] S. Bose, P. L. Knight, M. B. Plenio, and V. Vedral, Phys. Rev. Lett. **83**, 5158(1999).
- [6] M. B. Plenio, S. F. Huelga, A. Beige, and P. L. Knight, Phys. Rev. A **59**, 2468(1999).
- [7] G. Chimczak, Phys. Rev. A **71**, 052305(2005).
- [8] L. M. Duan and H. J. Kimble, Phys. Rev. Lett. **90**, 253601(2003).

- [9] L. M. Duan, M. D. Lukin, J. I. Cirac, and P. Zoller, *Nature(Lodon)* **414**, 413(2001).
- [10] C. Simon and W. T. M. Irvine, *Phys. Rev. Lett.* **91**, 110405(2003).
- [11] X. L. Feng, Z. M. Zhang, X. D. Li, S. Q. Gong, and Z. Z. Xu, *Phys. Rev. Lett.* **90**, 217902(2003).
- [12] X. B. Zou, K. Pahlke, and W. Mathis, *Phys. Rev. A* **68**, 024302(2003).
- [13] Z. Ficek and R. Tanaś, *quant-ph/0302124*(2003).
- [14] A Ekert, *Phys. Rev. Lett.* **67**, 661 (1991); A Beveratos, R Brouri, T Gacoin, André Villing, Jean-Philippe Poizat, and Philippe Grangier, *Phys. Rev. Lett.* **89**, 187901 (2002).
- [15] M. J. Bremner, et al., *Phys. Rev. Lett.* **89**, 247902(2002); X. Zou and W. Mathis, *Phys. Rev. A* **71**, 042334(2005).
- [16] M. J. Holland, D. F. Walls, and P. Zoller, *Phys. Rev. Lett.* **67**, 1716(1991); W. P. Schleich, *Quantum Optics in Phase Space* (Wiley-Vch, Berlin, 2001).
- [17] D. F. Walls and G. J. Milburn, *Quantum Optics* (Springer, Berlin, 1994).
- [18] H. M. Wiseman and G. J. Milburn, *Phys. Rev. A* **49**, 4110(1994).
- [19] D. Gunlycke, V. M. Kendon, and V. Vedral, *Phys. Rev. A* **64**, 042302(2001).
- [20] L. M. Duan, A. Kuzmich, and H. J. Kimble, *Phys. Rev. A* **67**, 032305 (2002).
- [21] K. Wootters, *Phys. Rev. Lett.* **80**, 2245 (1998).
- [22] T. Pellizzari, S. A. Gardiner, J. I. Cirac, and P. Zoller, *Phys. Rev. Lett.* **75**, 3788(1995).
- [23] S. J. Gu, H. Q. Lin, and Y. Q. Li, *Phys. Rev. A* **68**, 042330 (2003).
- [24] G. Ghimezak, R. Tanaś, and A. Miranowicz, *Phys. Rev. A* **71**, 032316(2005).

Figure Captions:

Fig. 1: Schematic diagram of the supposed system setup. Three two-level atoms are located in spatially separated microcavities which are connected via optical fibers. Two of the cavities are single-sided and one of them are double-sided.

Fig. 2: Entanglement of atom 1 and 2 versus time for A: (dotted line) $J_{31} = 0$, B: (solid line) $J_{31}=0.5$.

Fig. 3: Entanglement of A: (dotted line) atom 1 and 2 for $J_{12} = 4$, B: (solid line) atom 2 and 3 for $J_{23} = 4.1$ versus time.

Fig. 4: Entanglement of atom 1 and 2 versus time for local coupling strength $\Gamma = 0.2$.





